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## NLC ORIENTATIONAL INSTABILITY IN THE ABSORBED LIGHT WAVE FIELD WITH SPATIALLY MODULATED INTENSITY

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**Abstract** The influence of dipole phototransformed molecules on the orientational instability threshold of homeotropic NLC exposed to spatially modulated light is studied. The orientational spatial distribution of the director is calculated, in linear approximation, at the non threshold director reorientation. It is shown that the orientational instability threshold may significantly decrease (increase) and the optical nonlinearity constant may increase (decrease) by orders of magnitude as phototransformed molecules with typical value of permanent electric dipole appear in the nematics.

### INTRODUCTION

It is well known that nematic liquid crystals (NLC) possess giant optical nonlinear susceptibility which gives the opportunity to record dynamic holographic diffraction gratings<sup>1,2</sup>. In addition to optical orientation nonlinearity a large optical conformational nonlinearity appears in some NLC caused by the transition of the NLC molecules into long living phototransformed states. As the parameters of the phototransformed molecules (PM) differ from the parameters of the main NLC molecules the writing of the dynamic holographic gratings is enhanced by the conformational nonlinearity, too<sup>3</sup>.

The series of papers were published where the significant decrease (approximately by two orders of magnitude) of the threshold of the light induced Freedericksz transition was observed in NLC containing absorbing impurity molecules<sup>4-6</sup>. It was assumed that the observed anomalous optical reorientation is due to the interaction between the excited impurity molecules and the nematic host.

In our paper we suppose the appearance of PM in NLC under the action of light and the change of interaction between PM and main molecules. To take the later into account and realize the corresponding estimation we use the realistic assumption that PM, for example, gain electric dipole moment due to the phototransformation which is absent in the ground state of the molecules. We consider the influence of the dipole of

PM and the light intensity space modulation on the threshold value of light electric field and the efficiency of the dynamic gratings recording on the orientation optical nonlinearity. In the Section 1 the contribution to the free energy arising from dipole PM is obtained. As the light absorption coefficient and hence the PM concentration depend on the director orientation with respect to the light polarization the PM contribution will influence the director orientation in the light wave field. The influence of PM and finite anchoring energy on the director reorientation threshold is considered in Section 2. The director spatial distribution in the light field with intensity grating and the diffraction efficiency of gratings on conformation-orientational nonlinearity is studied in Section 3.

### **1. THE FREE ENERGY OF NLC WITH PHOTOTRANSFORMED MOLECULES**

Let us have the cell of homeotropically aligned NLC with the initial (undisturbed) orientation of director  $\vec{n}_0$  along the axis  $OZ$  and bounded by the planes  $z = 0$  and  $z = L$ . Two plane monochromatic light waves are incident on the cell making the angle  $2\theta$  between their wave vectors. The waves have equal amplitudes and are polarized along the same direction in the  $XOZ$  plane but their wave vectors are deflected symmetrically with respect to the  $XOZ$  plane:

$$\begin{aligned}\vec{E}_{1,2} &= \vec{E}_0 \cos(\omega t - \vec{q}_{1,2} \vec{r}) \\ \vec{E}_0 &= E_0 (\cos\beta, 0, -\sin\beta) \\ \vec{q}_{1,2} &= q(\cos\theta \sin\beta, \pm \sin\theta, \cos\theta \cos\beta)\end{aligned}\quad (1)$$

Here  $\beta$  is the angle between the wave polarization vector and the axis  $OX$ ,  $q$  is the wave vector value.

In consequence of interference in the cell volume the light wave field has the form

$$\begin{aligned}\vec{E}_{real} &= \frac{1}{2} [\vec{E}(\vec{r}) e^{-i\omega t} + \vec{E}^*(\vec{r}) e^{i\omega t}], \\ \vec{E}(\vec{r}) &= 2\vec{E}_0 \cos\left(\frac{\Delta q y}{2}\right) \exp\left\{i \frac{1}{2} q \cos\theta (x \sin\beta + z \cos\beta)\right\} \\ \Delta q &= 2q \sin\theta\end{aligned}\quad (2)$$

Let us further consider the case when owing to the light absorption the NLC molecules change their conformation and create the phototransformed states with lifetime  $\tau$ . We shall assume the characteristic times of the PM concentration changes are small in comparison with the director reorientation time. Then the PM concentration value  $c$

follows adiabatically the director change and may be found as a solution of the next equation:

$$\frac{\partial c}{\partial t} = -\frac{c}{\tau} + D \frac{\partial^2 c}{\partial y^2} + \delta \chi_{\alpha\beta} I_{\alpha\beta} (1-c) \quad (3)$$

where  $D$  is the PM diffusion coefficient,  $I_{\alpha\beta} = \frac{1}{2} E_\alpha E_\beta^*$  is the light intensity tensor,  $\chi_{\alpha\beta} = \chi_\perp \delta_{\alpha\beta} + \chi_a n_\alpha n_\beta$  is the tensor absorption coefficient averaged over the molecule long axis orientations,  $\chi_a = \chi_\parallel - \chi_\perp$ ,  $\chi_\parallel, \chi_\perp$  are the absorption coefficients for light polarized along the director and perpendicular to it,  $n_\alpha$  are the director components,  $\delta$  is the quantum efficiency of the NLC molecule phototransformation.

In stationary case the solution of equation (3) at  $c \ll 1$  is

$$c = \frac{\tau\delta}{1 + \tau D (\Delta q)^2} \left\{ \tau D (\Delta q)^2 \left[ \chi_\perp E_0^2 + \chi_a (\vec{n} \vec{E}_0)^2 \right] + \frac{1}{2} \left[ \chi_\perp |E|^2 + \chi_a (\vec{n} \vec{E})(\vec{n} \vec{E}^*) \right] \right\} \quad (4)$$

where

$$\begin{aligned} \chi_\parallel &= \tilde{\chi}_\parallel + \frac{1}{3} \tilde{\chi}_a (1 + 2S), \\ \chi_\perp &= \tilde{\chi}_\perp + \frac{1}{3} \tilde{\chi}_a (1 - S), \quad \chi_a = \chi_\parallel - \chi_\perp = \tilde{\chi}_a S \end{aligned} \quad (5)$$

$\tilde{\chi}_a = \tilde{\chi}_\parallel - \tilde{\chi}_\perp, \tilde{\chi}_\parallel, \tilde{\chi}_\perp$  are the absorption coefficients for light polarized along and perpendicular to the molecular long axis,  $S$  is the orientational order parameter. It should be noted that the sign of  $\tilde{\chi}_a$  and hence  $\chi_a$  strongly depends on the direction of the molecular transition dipole moment.

Taking into account the expression (2) for  $\vec{E}(\vec{r})$  one can see that the dynamical diffraction grating of the PM concentration with spatial period  $l = 2\pi / \Delta q$  appears in the NLC cell.

Assuming that due to phototransformation the PM gain permanent dipoles one can obtain the PM contribution to the NLC free energy. As the PM concentration  $c \ll 1$  we can neglect the interaction between the permanent dipoles but must take into account the interaction of the permanent dipoles with the induced dipoles of the neighbor NLC molecules.

The dipole moment induced at the molecule  $m$  by the electric field of all the PM permanent dipoles equals

$$\bar{P}^m = \hat{\alpha}^m(\Omega_m) \sum_n c_n \bar{E}^n(\bar{r}_{mn}, \Omega_n), \quad (6)$$

where  $\hat{\alpha}^m(\Omega_m)$  is the molecular polarizability tensor,  $\Omega_m$  denotes the angle coordinates of the molecule long axis,  $\bar{E}^n(\bar{r}_{mn}, \Omega_n)$  is the electric field vector of the permanent dipole of molecule  $n$  in the point placed on the neighboring molecule  $m$ ,  $\bar{r}_{mn} = \bar{r}_m - \bar{r}_n$ ,  $\bar{r}_n$  is the radius-vector of the molecule center of mass,  $c_n = 1$  if the PM is in the point  $\bar{r}_n$  and  $c_n = 0$  in the opposite case.

The interaction energy of the induced dipoles  $\bar{P}^m$  with the electric field which induces them is

$$U = -\frac{1}{2} \sum_m \hat{\alpha}^m(\Omega_m) \sum_n \sum_l \bar{E}^n(\bar{r}_{mn}, \Omega_n) \bar{E}^n(\bar{r}_{ml}, \Omega_l) \quad (7)$$

Averaging the expression (7) over the molecule orientation and the spatial distribution of the molecule center of mass, neglecting the terms proportional to  $c^2$  and replacing the summation over  $m$  by the volume integration one can obtain:

$$\bar{U} = -\frac{1}{2} \int c(\bar{r}) \alpha_{ii} \alpha_{ii} dV \quad (8)$$

where  $\alpha_{ii}$  are the main values of the NLC static polarizability tensor, index  $i$  denotes the Cartesian axes of a coordinate system fixed to the NLC director ( $OZ \parallel \bar{n}$ ). In this coordinate system

$$\alpha_{ii} = \sum_{n(\neq m)} \left\langle [E_i^n(\bar{r}_{mn}, \Omega_n)]^2 \right\rangle \quad (9)$$

$$\begin{aligned} \alpha_{ii} &= \frac{1}{4\pi} [(\varepsilon_{\perp}^0 - 1) + \varepsilon_a^0 \delta_{iz}], \\ \varepsilon_a^0 &= \varepsilon_{\parallel}^0 - \varepsilon_{\perp}^0 \end{aligned} \quad (10)$$

Here  $\langle \dots \rangle$  denotes orientational averaging,  $\varepsilon_{\parallel}^0, \varepsilon_{\perp}^0$  are the main values of the NLC static dielectric susceptibility tensor. In obtaining (8) we neglect the change of NLC director on the length of the permanent dipole-induced dipole interaction. In other words we neglect

the terms containing products of the type  $\frac{\partial n_i}{\partial x_j} \frac{\partial n_k}{\partial x_l}$  which lead to the dependence of

Frank's elastic constants on the PM concentration. This dependence is well-known and small enough so we do not consider it here.

Thus the free energy of the homeotropic NLC cell in the light wave field inducing the PM has the form

$$F = F_{elastic} + F_E + F_s + \bar{U} \quad (11)$$

$$F_{elastic} = \frac{1}{2} \int dV \left\{ K_1 (\text{div} \vec{n})^2 + K_2 (\vec{n} \text{rot} \vec{n})^2 + K_3 ([\vec{n} \text{rot} \vec{n}])^2 \right\} \quad (12)$$

$$F_E = -\frac{1}{16\pi} \int \varepsilon_{ij} E_i E_j^* dV \quad (13)$$

$$F_s = -\frac{1}{2} W \int (\vec{n} \vec{e})^2 ds, \quad W > 0 \quad (14)$$

Here  $F_{elastic}$  is the Frank's elastic energy,  $F_E$  is the energy of the light wave in NLC<sup>1</sup>,  $F_s$  is the energy of the director interaction with the cell surfaces<sup>7</sup>,  $\bar{U}$  is determined by the formulas (4), (8)-(10),  $\varepsilon_{ik} = \varepsilon_{\perp} \delta_{ij} + \varepsilon_a n_i n_k$  is the NLC dielectric susceptibility tensor at the light frequency,  $\varepsilon_a = \varepsilon_{\parallel} - \varepsilon_{\perp}$ ,  $\vec{e}_z$  is a unit vector along the easy orientation axis on the NLC surface ( $\vec{e}_z \parallel OZ$ ).

Now consider the NLC orientational instability with respect to the director deformation in the plane  $XOZ$  where the undisturbed director  $\vec{n}_0$  and the light field vector  $\vec{E}$  are placed. The equation for director may be obtained using the ordinary variational procedure for the system with free energy (11). Below we shall consider it for the cases of threshold and non threshold director reorientation.

## 2. THE THRESHOLD DIRECTOR REORIENTATION

In this case  $\varepsilon_a > 0$ , the angle  $\beta = 0$  and the field vector  $\vec{E}_0 \perp \vec{n}_0$  ( $\vec{E}_0 \parallel OX$ ). It is convenient to present the director in the following form  $\vec{n} = \sin \phi(y, z) \vec{i} + \cos \phi(y, z) \vec{k}$ , where  $\vec{i}, \vec{j}, \vec{k}$  are the unit vectors of the Cartesian coordinate frame. The angle  $\phi$  of the director deviation from its undisturbed state does not depend on the  $x$  coordinate because of the homogeneity of the system in the direction  $OX$ .

In order to obtain the threshold value of the light wave electric field, it is sufficient to keep in the variational equation only the terms linear in  $\phi$ . Then in one constant approximation this equation takes the form:

$$K \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) + B E_0^2 \cos^2 \left( \frac{1}{2} \Delta q y \right) \phi = \eta \frac{\partial \phi}{\partial t} \quad (15)$$

where

$$B = \frac{\varepsilon_{\perp}}{2\pi\varepsilon_{\parallel}}(\varepsilon_a + \Delta)$$

$$\Delta = \frac{1}{2}\tau\delta\chi_a[(a_{xx} + a_{yy})(\varepsilon_{\perp}^0 - 1) + a_{zz}(\varepsilon_{\parallel}^0 - 1)]$$
(16)

The term on the right-hand side of equation (15) appears if one takes into account the relaxation processes neglecting the connection between the director and hydrodynamical motions,  $\eta_0$  is the relaxation constant ( $\eta_0 \sim 10^{-2} - 1P$ ). Here at the derivation of formulas (15), (16) and also in the next section to simplify the expressions we neglect the terms connected with the PM diffusion supposing the inequality  $D(\Delta q)^2 \ll \tau^{-1}$  is fulfilled.

The boundary conditions for equation (15) have the form:

$$\left(\frac{\partial\phi}{\partial z} + \frac{W}{K}\phi\right)_{z=L} = 0$$

$$\left(\frac{\partial\phi}{\partial z} - \frac{W}{K}\phi\right)_{z=0} = 0$$
(17)

The solution of equation (15) must be even and periodic with respect to the coordinate  $y$  in accordance with the character of the light field (2) disturbing the NLC director. We shall seek it using the method of separating the variables. Then the boundary condition requirements lead to the following expression for  $\phi$ :

$$\phi(y, z, t) = \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \phi_{nr} \left( \frac{K\mu_n}{W} \cos \mu_n z + \sin \mu_n z \right) \cdot$$

$$ce_r \left( \frac{1}{2} \Delta q y, -\frac{4BE_0^2}{K(\Delta q)^2} \right) \exp(\Gamma_{nr} t),$$
(18)

where the function  $ce_r(\xi, \eta)$  is the even solution of Mathieu's equation

$$\frac{d^2 f}{d\xi^2} + (a - 2s \cos 2\xi) f = 0,$$
(19)

corresponding to the eigenvalue  $a_r(s)$ ,

$$\Gamma_{nr} = \frac{1}{\eta} \left[ 2BE_0^2 - K\mu_n^2 - \frac{1}{4}(\Delta q)^2 Ka_r \left( -\frac{4BE_0^2}{K(\Delta q)^2} \right) \right] \quad (20)$$

$\phi_{nr}$  are integration constants,  $\mu_n$  is the nontrivial solution of the equation

$$\tan \mu L = \frac{2WK\mu}{(K\mu)^2 - W^2}, \quad (21)$$

As it is seen from the solution (18) the director disturbance will be growing with

time if some of the values  $\Gamma_{nr} > 0$ . If  $\chi_a > 0$ , then  $B > 0$ ,  $Q = -\frac{4B}{K(\Delta q)^2} E_0^2 < 0$  and  $a_0(q) < a_1(q) < a_2(q) < \dots$ <sup>8</sup>. In this case evidently the threshold value of the amplitude  $E_0$  can be found from the condition  $\Gamma_{10} = 0$ , where  $\mu_1$  is the lowest (nontrivial) root of equation (21). If we denote by

$$E' = \left( \frac{K}{2B} \right)^{1/2} \mu_1, \quad (22)$$

the equation for the threshold value  $E_0$  can be written in the form

$$\left( \frac{E_0}{E'} \right)^2 = 1 + \left( \frac{\Delta q}{2\mu_1} \right)^2 a_a \left[ -2 \left( \frac{\mu_1}{\Delta q} \frac{E_0}{E'} \right)^2 \right] \quad (23)$$

In general case this equation can only be solved only numerically. Let us consider some limiting cases when the solution of the equation (23) have an analytical form.

Let  $\Delta q$  be large enough, so that  $|Q| \ll 1$ . Then  $a_0(Q) \approx -\frac{1}{2}Q^2$  and as it follows from (23)

$$E_{0th} = E' \left[ 1 - \left( \frac{\mu_1}{2\Delta q} \right)^2 \right] \quad (24)$$

If the quantity  $\Delta q$  is small, so that  $|Q| \gg 1$  then  $a_0(Q) \approx -2Q - 2|Q|^{1/2}$  and one can get from (23)

$$E_{0th} = \frac{E'}{\sqrt{2}} \left( 1 + \frac{\Delta q}{2\mu_1} \right) \quad (25)$$



In the case of the strong anchoring ( $W = \infty$ )  $\mu_1 = \frac{\pi}{L}$  and taking into account that

$\Delta q = \frac{2\pi}{l}$ , where  $l$  is the spatial period of the light field intensity, one can get the condition  $|Q| \ll 1$  is equivalent to the inequality  $l \ll L$  (and analogously in the case of opposite inequality). If  $W = \infty$ ,  $\Delta q = 0$  and the PM are absent one can obtain from (25) the well known result for the threshold of light induced Freedericksz transition in the homogeneous light field.

Thus the expressions (22)–(25), and (16) determine dependence of the threshold value of the light wave electric vector on the spatial period of the light field intensity, values of the PM parameters and anchoring energy (parameter  $\mu_1$ ).

As it follows from (16) the appearance of dipole PM leads (from the formal point of view) to the renormalization of the anisotropic part of NLC dielectric susceptibility by the terms to be proportional to the absorption coefficient anisotropy and the interaction parameters between PM and NLC. To obtain the director space distribution just above the threshold it is necessary to leave in the variational equations (15)–(17) the terms nonlinear in  $\phi^1$ . But it is beyond the scope of this paper.

In order to estimate the contribution of PM to the threshold value  $E_{th}$  we substitute in the formula (9) the field value of the electric dipole  $\vec{d}_n$

$$\vec{E}^n = \frac{3(\vec{d}_n \vec{r}_{mn}) \vec{r}_{mn}}{r_{mn}^5} - \frac{\vec{d}_n}{r_{mn}^3} \quad (26)$$

and perform the averaging over the molecular long axis orientations. Then we shall get the terms renormalizing  $\epsilon_a$  in the form

$$\Delta = \chi_a \tau \delta \left[ (3\gamma + \gamma') (\epsilon_{\perp}^0 - 1) + \epsilon_a^0 \left( \gamma + \frac{2}{3} \gamma' \left( S + \frac{1}{2} \right) \cos^2 \psi \right) \right], \quad (27)$$

where the positive quantities  $\gamma \sim \gamma' \sim \frac{d^2}{R^6}$ ,  $R$  is the average distance between the NLC molecules,  $d$  is the PM permanent dipole value.

It is seen that the sign of the quantity  $\Delta$  depends on the absorption coefficient anisotropy. Since  $\chi_a > 0$  the threshold value of the electric field (see formulas (22), (16)) will decrease. From (23) it follows that  $\Delta$  is of the same order of magnitude as  $\chi_a \tau \delta \gamma \epsilon_{\perp}^0$ . To estimate  $\Delta$  one can take into account that the absorption coefficient

$\chi\left(\frac{J}{m^3s}\right) = \frac{c_0}{4\pi} \frac{\alpha}{N\hbar\omega}$ , where  $c_0 = 3 \cdot 10^8 \frac{m}{s}$ ,  $N(m^{-3})$  is the NLC molecule concentration,  $\alpha$  is the absorption coefficient in  $m^{-1}$ ,  $\omega$  is the absorption frequency. Let us suppose, for example, the PM are created at the light absorption with wave length  $\lambda = 0.44 \mu m$ ,  $\tau = 0.1s$ ,  $\alpha = 1000 m^{-1}$ ,  $N = 3 \cdot 10^{27} m^{-3}$  <sup>9</sup>. Putting  $\delta = 0.1$ ,  $R = 10^{-9} m$ ,  $\chi_a \sim \chi$  we obtain  $\Delta \sim 10^{40} \epsilon_{\perp}^0 d^2$ .

Thus, if we assume for the PM permanent dipole the typical value  $d = 1Db$  then the term renormalizing  $\epsilon_a$  is of the order of  $10^4 \epsilon_{\perp}^0 \gg \epsilon_a$ . As the result the threshold value of the electric vector  $\vec{E}_0$  will decrease considerably (by orders of magnitude). The creation of the PM with large enough permanent dipole under the action of light might have been the reason of the considerable decreasing of the director reorientation threshold observed in <sup>3,4</sup>.

It is interesting to note, that in the case of negative absorption coefficient anisotropy ( $\chi_a < 0$ ), the parameter  $B$  decreases and the threshold value of  $\vec{E}_0$  will increase or even director reorientation may absent at all if  $B < 0$ .

The formulas obtained above give also the temperature dependence of the threshold value,  $E_{0th}$ . But this dependence is too complicated and must be analyzed numerically.

### **3. THE NON THRESHOLD DIRECTOR REORIENTATION**

Now we write down the director in the same form as in Section 2, but let us suppose that the electric field  $\vec{E}_0$  makes the acute angle  $\beta$  with the initial director  $\vec{n}_0$  ( $0 < \beta < \pi/2$ ). Director reorientation process in this case will occur without threshold. Consider the stationary director distribution when its deviation  $\phi(y, z)$  from the initial state is small. We suppose for simplicity strong director anchoring with the cell surfaces ( $W = \infty$ ) and neglect as before the PM diffusion. Then the equation following from the condition of free energy minimum and linearized with respect to  $\phi$  takes the form:

$$K \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) + B \cos 2\beta E_0^2 \cos^2 \left( \frac{1}{2} \Delta q y \right) \phi = \frac{1}{2} B \sin 2\beta E_0^2 \cos^2 \frac{\Delta q y}{2} \quad (28)$$

with the boundary conditions  $\phi = 0|_{z=0,L}$ .

Using the method of separation of variables we obtain the solution of equation (28) in the form:

$$\phi(y, z) = \sum_{n=1,3,\dots} f_n\left(\frac{\Delta q y}{2}\right) \sin \frac{\pi n}{L} z \quad (29)$$

where functions  $f_n(\xi)$  are solutions of the following non homogeneous Mathieu's equation:

$$\frac{d^2 f_n(\xi)}{d\xi^2} + (a_n - 2p \cos 2\xi) f_n(\xi) = b_n (1 + \cos 2\xi) \quad (30)$$

Here

$$\begin{aligned} a_n &= \frac{4}{(\Delta q)^2} \left[ \frac{B \cos 2\beta E_0^2}{2K} - \left( \frac{\pi n}{L} \right)^2 \right] \\ p &= -\frac{E_0^2}{K(\Delta q)^2} B \cos 2\beta \\ b_n &= \frac{4BE_0^2}{n\pi K(\Delta q)^2} \sin 2\beta \end{aligned} \quad (31)$$

Taking into account that equation (30) is periodic and even with respect to the variable  $\xi$ , we try its partial solution in the series form

$$f_n(\xi) = \sum_{m=0}^{\infty} C_{2m}^n \cos(2m\xi) \quad (32)$$

On substitution of the series (32) into equation (30) one obtains the infinite system of equations for the coefficients  $C_{2m}^n$ . It can be proved that the ratio

$\frac{C_{2m}^n}{C_{2m-2}^n} = O\left(\frac{1}{m^2}\right)$ , thus the series (32) converges absolutely and uniformly, and it makes possible to restrict the consideration to the finite number of equations for  $C_{2m}^n$  with any desired accuracy.

The cubic optical nonlinearity parameters  $\eta$ , that determine the diffraction efficiency of director dynamic gratings generated in the NLC cell are proportional to the parameters  $C_{2m}^n$ . If we restrict our consideration, for example, to the terms in (32) with  $m \leq 2$ , then director distribution along OY axes will be approximately a superposition of

two cosine gratings with periods of  $l_1 = \frac{2\pi}{\Delta q}$  and  $l_2 = \frac{\pi}{\Delta q}$ . Nonlinearity parameter  $\eta$  corresponding to the grating with biggest period in this case takes the form

$$\eta_n \sim \frac{C_2^n}{E_0^2} = \frac{\frac{b_n}{a_n - 4} \left(1 + \frac{p}{a_n}\right)}{1 - 2 \frac{p^2}{a_n(a_n - 4)} \frac{a_n - 8}{a_n - 16}} \quad (33)$$

Substituting the values of  $a_n$ ,  $p$ ,  $b_n$  it is not difficult to obtain the dependence of nonlinearity parameter on the grating period and parameters, characterizing the contribution of PM. Letting, for example,  $\beta = \frac{\pi}{8}$  and considering the fields  $E_0 \ll E'$  we obtain for most intensive director mode with  $n = 1$  the following estimation

$$\eta_1 \sim \frac{C_2^1}{E_0^2} \approx \frac{1}{2\pi} \frac{\left(\frac{l_1}{L}\right)^2}{\left(\frac{l_1}{L}\right)^2 + 4} \frac{1}{(E')^2} \quad (34)$$

Since PM appearance can decrease (if  $\chi_a > 0$ ) the value of  $E'$  by some orders (see Sec. 3), then the nonlinearity parameter of diffraction grating in the case of conformation-orientational nonlinearity must be significantly greater (proportionally to  $E'^{-2}$ ) than in the case of pure ( $c = 0$ ) orientational nonlinearity. If  $\chi_a < 0$ , but  $B$  remains positive, then the nonlinearity parameter of diffraction grating will decrease. In the case of negative both  $\chi_a$  and  $B$ , the nonlinear parameter will increase at  $|\Delta| > 2\varepsilon_a$  and will exceed the value of pure orientational nonlinearity parameter.

## CONCLUSIONS

If PM gain permanent electric dipole moment at the phototransformation and the light absorption coefficient anisotropy  $\chi_a > 0$  ( $\chi_a < 0$ ) then the NLC orientation instability threshold in the light wave field may significantly decrease (increase) at the typical values of PM dipole moment ( $\approx 1Db$ ). The director anchoring energy as well as the spatial period of the incident light wave intensity have influence on the threshold value.

In the case of non threshold director reorientation the director space distribution is a superposition of gratings on conformation-orientational nonlinearity. The diffraction

efficiency of these gratings may significantly change in comparison with the diffraction efficiency of gratings on pure orientational nonlinearity depending on the sign of absorption coefficient anisotropy and the value of quantity  $\Delta$ .

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### **REFERENCES**

1. B.Ya. Zel'dovich, N.V. Tabiryan, Yu. S. Chilingaryan, ZhETF, **81**, 72 (1981)
2. B.Ya. Zel'dovich and N.V. Tabiryan, ZhETF, **82**, 1126 (1982)
3. S.G. Odulov, Yu.A. Reznikov, M.S. Soskin, A.I. Khizhnyak, Sov.Phys. JETP, **55**, 354 (1982)
4. I. Janossy, A.D. Lloid and B.S. Wherrett, Mol.Cryst.Liq.Cryst. **179**, 1 (1190)
5. I. Janossy, A.D. Lloid, Mol.Cryst.Liq.Cryst. **203**, 77 (1991)
6. I. Janossy, L. CsillagA.D. Lloid, Phys. Rev. A, **44**, 8410 (1991)
7. A. Rapini, M. Papolar, J.Phys. Colloq. **30**, 54 (1969)
8. Handbook of Mathematical Functions, edited by M. Abramowitz and I.A. Stegun, (National Bureau of Standards, 1964)
9. I.P. Pinkevich, Yu.A. Reznikov, V.Yu. Reshetnyak et. al. Ukr. Fiz. Zhurn., **32** 1216 (1987)